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AN EXPERIMENTAL STUDY OF HEAT AND MASS TRANSFER ACROSS A DIFFUSIVE INTERFACE

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NOMENCLATURE

C_P, specific heat;
D_M, mass diffusivity;
D_T, thermal diffusivity;
F_S, flux of solute through the interface;
g, gravitational acceleration;
H, heat flux across a diffusive interface;
H_P, heat flux from a heated solid plate into the deep fluid layer, defined in equation (1);
k, thermal conductivity;
Pr, Prandtl number;
Ra, Rayleigh number;
R_ρ, stability parameter $= \dfrac{\beta\,\Delta S}{\alpha\,\Delta T}$;
$R_{\rho C}$, stability parameter at which the transition from the constant to the variable regime takes place;
$R_{\rho 0}$, stability parameter at which heat flux across the diffusive interface is zero according to Linden and Shirtcliffe's analysis [see equation (4)];
S, solute concentration (g of solute per 100 g of solution).

Greek symbols
α, volume expansivity $= -\dfrac{1}{\rho}\left(\dfrac{\partial\rho}{\partial T}\right)_S$;
β, $= \dfrac{1}{\rho}\left(\dfrac{\partial\rho}{\partial S}\right)_T$;
ρ, density;
ΔS, concentration difference between layers;
ΔT, temperature difference between the centers of the layers;
τ, diffusivity ratio $= D_M/D_T$, or more generally the ratio of the diffusivity of the stabilizing component to that of the destabilizing component;
ν, kinematic viscosity.

INTRODUCTION

THE STUDY of heat and mass transfer across a diffusive interface has recently received much attention [1–6]. Originally, Turner [2] performed an experiment in which a sharp interface was produced between two different density layers of an aqueous solution of common salt, with heat applied to the lower layer. However, except for Shirtcliffe [3] who used sugar and salt as the diffusing components ($\tau = 1/3$ compared with $\tau = 1/100$ for heat and salt), all others used salt and heat as the stabilizing and destabilizing components, respectively, because of their importance in the study of the vertical transport of salt and heat in the ocean [7, 18]. In other areas of engineering interest, such as the large-scale storage of cryogenic fluid [9] and solar ponds [10], it has recently been found necessary to investigate cases of double-diffusive convection in which the stabilizing component is other than salt with heat as the destabilizing component. This paper reports the results of an experimental investigation in which as the stabilizing component three kinds of aqueous solutions [common salt, copper sulfate ($CuSO_4$) and hydrochloric acid (HCl)] have been used to produce diffusive interfaces in order to extend the applicability of previous works.

EXPERIMENT

The experimental method, which is essentially the same as Turner's, begins with the creation of a sharp density interface in a container followed by the measurement of the heat and mass transfer across the diffusive interface under application of heat to the lower layer. The container is an acrylic cylindrical tank (I.D. = 291 mm, height = 320 mm) insulated on the side wall and on the top by styrofoam pads. The bottom of the tank is immersed in a constant temperature bath. The location of the interface is at the midpoint of the tank depth. Twelve thermocouples were placed inside the tank to measure the temperature of both upper and lower layers. In order to measure the concentration of the stabilizing component, a small sample of liquid was extracted and its refractive index was read through a refractometer having the capability of analysing 1/1000% difference in concentration which gave the refractive index at 20°C. Since the total amount of the stabilizing component in the tank was known, the concentration measurement was made only for the upper layer to avoid disturbance of the interface by sampling probes. The data consisted of temperature and concentration of the stabilizing component as a function of time with a measuring interval of ~10 min. As is the case with this investigation, all the experiments so far have been the quasi-steady, run-down type. The time-dependent effects of this type of experiment have been discussed by Shirtcliffe [11], and all the data reported here are taken after convection is fully established in both layers. Table 1 lists the properties of the aqueous solutions pertaining to the transfer mechanism

Table 1. Properties of the aqueous solutions and the experimental range covered by this investigation

	HCl	NaCl	$CuSO_4$
Concentration [wt%]	0.1–3.5	0.1–2.5	0.1–2.5
Temperature [°C]	5–30	5–40	5–35
Applied heat flux [cal/cm² · s]	1.5×10^{-2}–9×10^{-2}	1.5×10^{-2}–9×10^{-2}	1.5×10^{-2}–9×10^{-2}
Ra	10^7–10^8	10^7–10^8	10^7–10^8
Pr	5–12	5–12	5–12
τ	0.021 ± 0.005	0.011 ± 0.004	0.003 ± 0.001

across the diffusive interface as well as the experimental range. As can be seen, the diffusivity ratio, τ, ranges from 0.021 of heat–HCl case to 0.003 of heat–$CuSO_4$ case. Since a fairly large number of liquid substances falls into the range of both τ and Pr, which this experimental investigation covers, we believe these results will apply to the cases in which the stabilizing component is other than those we used.

RESULTS AND DISCUSSION

Originally Turner [2] proposed the following relationship for a heat flux across a diffusive interface, H, based on dimensional arguments.

$$H/H_P = f(R_\rho, \tau)$$

where (1)

$$H_P = 0.085k\left(\frac{\theta\alpha}{\nu D_T}\right)^{1/3}(\Delta T)^{4/3}$$

Turner's experimental results for a heat–NaCl diffusive interface (i.e. a fixed value of τ) can be represented by the following empirical equation [12].

$$f(R_\rho, \tau \approx 0.011) = f_1 = 3.8R_\rho^{-2}. \tag{2}$$

On the other hand, Marmorino and Caldwell [6] from their own experiment suggested the following form for the heat–NaCl case:

$$f(R_\rho, \tau = 0.011) = f_2 = 0.101\exp\{4.6\exp[-0.54(R_\rho - 1)]\}. \tag{3}$$

Analytically, Linden and Shirtcliffe [11] recently used a thermal-burst model proposed by Howard [13] to obtain

$$f(R_\rho, \tau) = f_3 = \pi^{-1/3}\frac{(1-\tau^{1/2}R_\rho)^{4/3}}{(1-\tau^{1/2})^{1/3}}. \tag{4}$$

It should be noted that equation (4) predicts zero heat flux at $R_\rho = R_{\rho 0} = \tau^{-1/2}$. Figure 1 shows the results of heat-flux measurement for the heat–NaCl case along with equations (2)–(4) with $\tau = 0.011$. The data agrees closely with both equations (2) and (3) for $1 < R_\rho < 4$, but confirms the Marmorino and Caldwell results [equation (3)] that at higher values of the stability parameter the curve tends to flatten. Figure 2 shows the measured heat flux of both heat–HCl and heat–$CuSO_4$ cases with equation (3) (heat–NaCl curve) and two curves of equation (4) with $\tau = 0.003$ and 0.021 for heat–$CuSO_4$ and heat–HCl cases respectively, clearly revealing the difference in heat flux among the three diffusing components. The Linden and Shirtcliffe's analysis, equation (4), agrees reasonably well with the data for the intermediate values of R_ρ, but our data continue to show an approximately constant flux beyond the predicted $R_{\rho 0}$.

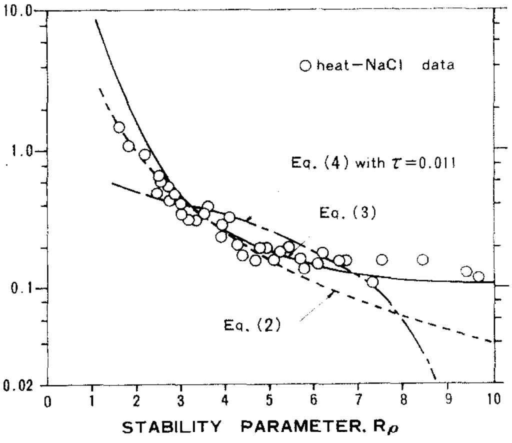

FIG. 1. Measured heat flux vs stability parameter for the heat–NaCl case.

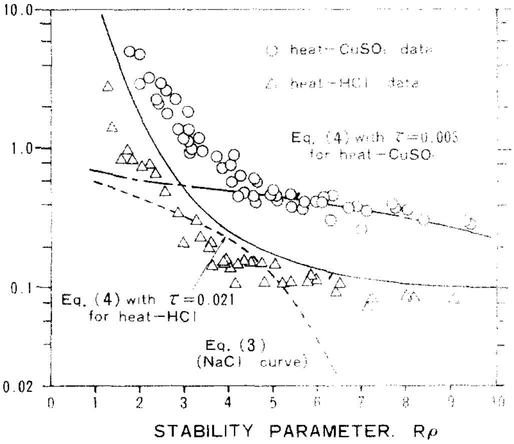

FIG. 2. Measured heat flux vs stability parameter for the heat–HCl and heat–$CuSO_4$ cases.

Figures 3 and 4 summarize the change of the buoyancy flux ratio, $\rho c_P(\beta F_S/\alpha H)$, with the stability parameter. They verify the existence of the variable (flux ratio) and the constant (flux ratio) regimes regardless of the substance for mass diffusion. As for the comparison between our heat–NaCl experiment and that of Turner shown in Fig. 3, the value of flux ratio at the constant regime agrees well within experimental error. Figure 4 shows that the value of $R_{\rho c}$ depends on the component for mass diffusion. The most surprising finding is that the flux ratio at the constant regime does not increase with $\tau^{1/2}$ as predicted by Linden and Shirtcliffe's analysis [11], and that, on the contrary, the heat–$CuSO_4$ experiment, which has the lowest value of τ of the three, registered the highest flux ratio at the constant regime.

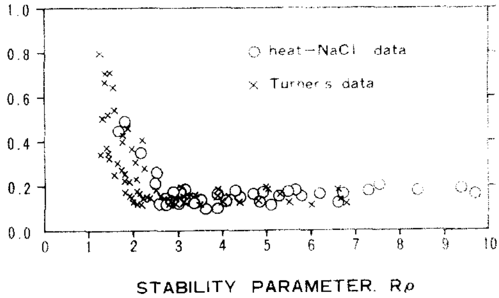

FIG. 3. Experimental results of the buoyancy flux ratio vs stability parameter for the heat–NaCl case.

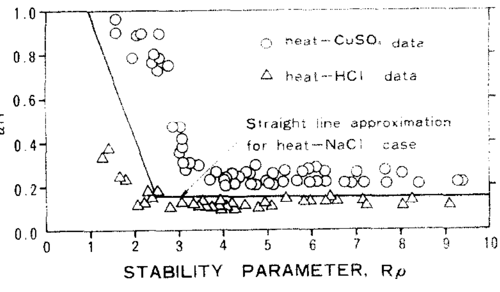

FIG. 4. Experimental results of the buoyancy flux ratio vs stability parameter for heat–HCl and heat–$CuSO_4$ cases.

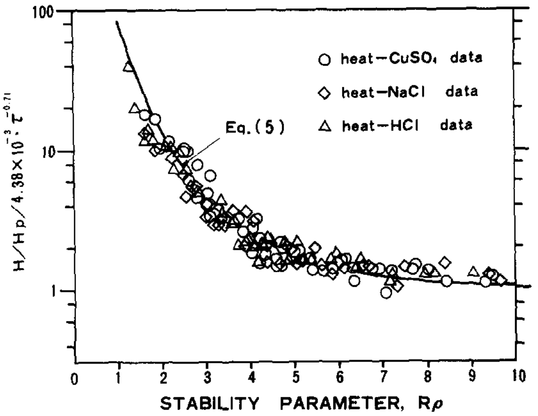

FIG. 5. Comparison between experimental data and the proposed empirical correlation for heat flux across a diffusive interface.

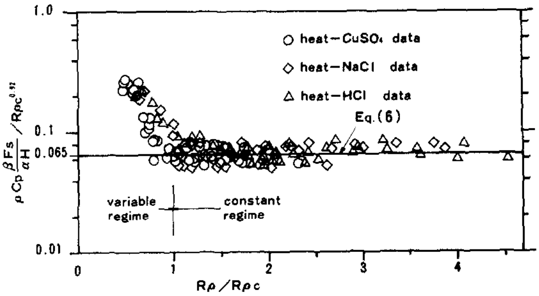

FIG. 6. Comparison between experimental data and the proposed empirical correlation for the buoyancy-flux ratio at the constant regime.

The reason for the existence of the variable regime and the discrepancy between the measured heat flux and Linden and Shirtcliffe's analysis at the high R_ρ range has been attributed to thermally-induced convection effect [6, 11], and need not be elaborated here. Linden and Shirtcliffe's analytical result that the buoyancy flux ratio at the constant regime should be equal to the square root of the diffusivity ratio, has so far been supported by both Turner's experimental results for the heat–NaCl case (measured flux ratio = 0.15, $\tau^{1/2}$ = 0.11) and Shirtcliffe's sugar–NaCl case (measured flux ratio = 0.600, $\tau^{1/2}$ = 0.577). On the other hand, Marmorino and Caldwell [6] measured a flux ratio at the constant regime as high as 0.4 for a heat–NaCl case when the applied heat flux is two orders of magnitude lower than both Turner's range ($1.8 \times 10^{-2} \sim 5.5 \times 10^{-2}$ cal/cm$^2\cdot$s) and this investigations range ($1.5 \times 10^{-2} \sim 9 \times 10^{-2}$ cal/cm$^2\cdot$s). Even with this uncertainty in the measured values of the flux ratio, the present experimental results clearly show the general behavior of the heat and mass flux across a diffusive interface when the destabilizing component is heat and the diffusivity ratio is varied. And based on our experimental results we obtained the following empirical equations.

$$H/H_P = 4.38 \times 10^{-3} \cdot \tau^{-0.71} \cdot \exp\{4.6 \exp[-0.54(R_P - 1)]\}, \quad (5)$$

$$\rho C_P \frac{\beta F_S}{\alpha H} = 0.034 \cdot \tau^{-0.33}$$

for (5)

$$R_\rho > R_{\rho C} = 0.49 \cdot \tau^{-0.38} \quad \text{(i.e. constant regime).}$$

Figures 5 and 6 show the comparison between our experimental data and the proposed empirical correlations, equations (5) and (6).

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